Critical raw materials for the energy transition

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Abstract

Growing evidence of the relative intensity in critical raw materials (CRM) of low-carbon technologies, attracts attention in policy debate. Yet, integrated assessment models, currently used in policy and academic debate on climate change mitigation policies, rule out any role of CRM as inputs to specific equipment in the energy transition, hereafter dubbed green capital. This article presents a model of the energy transition, where the policy is chosen to respect a carbon budget while minimizing the cost of climate policy, with green capital either embedding scarce minerals, or based on a relatively expensive backstop technology. We find that the smaller is the available stock of CRM, the lower welfare and the slower the fossil phase-out. We also show how abstracting from the scarcity of CRM may be severely misleading in designing climate policy. Finally, we highlight the potential role of recycling CRM in easing the energy transition. We find that the lower the cost of recycling, the slower the exhaustion of the CRM.

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Contents

1 Introduction

The high technological and economic importance of Critical Raw Materials (CRMs)[1] combined with geopolitical issues has led to increasing attention for CRMs used in renewable energy. It is within this framework that the French Secretary of State for Ecology, Ms. Brune Poirson, launched, on February 22, 2019, the work to develop a programming plan for low-carbon transition mineral resources. Indeed, to build the energy infrastructure essential to achieving its greenhouse gas emission reduction targets, France will need to mobilize more mineral resources, some of which may be critical.This observation is now widely shared, as evidenced by the works of the UN International Resource Panel (IRP, 2020), the World Bank (Arrobas *et al.*, 2017), and the European Commission (COM, 2017). The alternative would be to reduce the dependence on CRMs, through recycling or the discovery other technologies that do not rely on CRMs.

In this paper, we investigate how the timing of the energy transition is affected by CRMs while a very costly backstop technology also exists. The size of the stock of CRMs may speed-up or slow down the transition such that the time when fossils are abandoned may be sooner or later. In addition, the result may critically depend on whether recycling may effectively be used to release the stress imposed by CRMs on the energy transition.

A material is labeled as critical not on the basis of its reserve but on the basis of geological and economic factors. In 2017, the EU published its third list of critical materials with 27 CRMs based on a refined methodology (to be revised again in 2020). The two main criteria used by the European Commission to determine the criticality of a raw material are the economic importance and the supply risk. Economic importance refers to how crucial a material is for the EU economy in terms of end applications and added value of the corresponding EU manufacturing sectors. Supply risk is measured in terms of the concentration of primary supply from commodity producing countries, taking into account their governance performance and trade aspects (based on the EU import balance).

The work of the French Low Carbon Transition Mineral Resources Programming Plan focuses on 4 major families of low carbon technologies: photovoltaic, stationary storage and networks (including smart grids), low carbon mobility and wind power. The objective is to compare, within each of these families, the technologies that are mature or likely to be mature in the next 10 years with regard to the mineral resource needs they mobilize and the associated economic, geopolitical, environmental, health and social issues, and the industrial opportunities they may present for companies throughout their value chain. For instance, it can be noted that 90% of the current solar cell market is based on silicon. The consumption of silicon is about 10-15 g/Wp solar generation. Silicon is a critical raw material as it is of significant economic importance for several sectors beyond solar power generation. There is no real issue regarding the availability of the reserve as there is enough "solar grade" quartzite in Europe for 10 years of production, but a large proportion of crystalline PV production is carried out in China, with a high degree of vertical integration, which creates a certain industrial vulnerability. Furthermore, it can be noted that silver is also significantly

[1]Distinction must be made between CRMs and rare earth elements. The European Commission (2017) defines CRMs as "raw materials of high importance for the EU economy and whose supply is associated with a high risk". The same study defines them as "a set of 15 elements of the lanthanide series and two other elements: scandium and yttrium". Despite their name, rare earths are relatively abundant in the earth's crust. However, because of their geochemical properties, rare earths are generally dispersed and are not often concentrated in minerals.

mobilized in crystalline technologies. Indeed, PV is the third largest user of silver and there is a real tension on the availability of the material, whereas there is no possibility of substitution in the short term (for metallisation, copper has been unsuccessfully studied and moreover, silver is not only used in metallisation). In addition, the European wind energy sector is permanently dependent on Chinese supplies of neodymium (90%) and dysprosium (99%).

There exists a very abundant literature of macro-dynamic models à la Hotelling where the renewable energy shares the characteristics of the backstop technology (see Hoel and Kverndokk, 1996, and Tahvonen, 1997): it consists in an abundant and steady flow available with certainty, at a unit cost higher than the unit extraction cost of fossil energy. In the simplest version of this representation of the energy transition, the unit extraction cost of fossil fuels rises over time, following a Hotelling rule and ultimately reaching the cost of the backstop at time of the switch to clean renewable energy. In our model, renewable energy differs from the backstop as the transition between non renewable and renewable energy sources requires adjustment costs over the production capacity of renewable as in Amigues et al. (2015). In addition, we assume that investment in such a production capacity requires minerals. The consideration of resources in the energy transition as well as the recycling of these resources is rare and very recent in the literature. In Fabre et al. (2020), agents value energy services which result from a combination of energy provided by a renewable source and the combustion of a non renewable fossil resource that are not perfect substitutes. When a unit of mineral resources is embedded in the equipment and infrastructure used to produce energy from renewable sources, it supplies a flow of energy services until the end of the life cycle of the equipment, when it adds to the stock of secondary mineral resources that can be recycled. The recycling process is constant, costless and exogenous. Climate damages are introduced in a 2-period version of the model. In such a framework, they show that the reliance of renewable energy on minerals favors abundant and early investment in green capital for the production of renewable energy, given that minerals embedded in specialized green capital can be recycled, as opposed to fossil resources burned for energy production. Recycling is also considered in a dynamic model in Lafforgue and Rouge (2019) where the use of a non-renewable resource yields waste that can be recycled once investment in R&D is sufficient for the recycled material quality to meet a certain standard. They show that recycling urges a more intense exploitation of the resource. In our model, part of the green capital can be recycled at a cost that depends on both the size of the treated secondary stock and the recycled fraction. Such a recycling opportunity potentially loosens the constraint on minerals. We study the optimal timing of recycling and how the recycling opportunity affects the speed of the energy transition.

We extend the model of Pommeret and Schubert (2019) to integrate the use of scarce resources in the energy transition. Pommeret and Schubert (2019) propose a stylized dynamic deterministic model of the optimal choice of the electricity mix (fossil and renewable), where fossil energy, e.g. coal, is abundant but emits CO2, while renewable energy, e.g. solar, is variable and clean. This model distinguishes electricity consumed at night from that consumed during the day in order to study the problem of intermittent renewables but to focus on the consequences of the scarcity of critical resources needed for energy transition rather than intermittency, we will not distinguish between these two types of consumption here.[2] Coal and solar energy are assumed to be available at zero variable cost, in order to focus on the issue of critical resources. It is also assumed that, at

[2]However, it will be necessary to reintroduce this distinction if we wish to extend our study to the resources that go into the composition of batteries, which themselves are necessary to manage the intermittency of renewables.

the beginning of the planning horizon, coal-fired plants already exist, so that there is no capacity constraint on fossil-fueled electricity generation. On the contrary, existing solar capacity is small and requires investments in green capital that embeds a given amount of critical resources. Alternatively, a relatively expensive backstop technology can be used to build-up green capital, which represents the CRMs-free alternative.

The centralized program is solved under the constraint of a carbon budget that cannot be exceeded. There is therefore a trade-off between on the one hand fossil energy which is expensive to use in terms of CO_2 emissions and on the other hand, renewable energy which is expensive because it requires costly investment in specialized equipment, hereafter dubbed green capital, which relies on critical mineral resource that is being depleted or a relatively costly backstop technology.

We analyze the optimal trajectories considering different potential successions of phases: minerals may be exhausted before or after the carbon budget is reached, depending on the relative size of the two stocks. In any case, it is optimal not to use any fossil fuels at all once the carbon budget is exhausted.

We study how the criticality constraint affects the dynamics of the energy transition through the accumulation of green capital. The speed of this transition undoubtedly depends on the relative strength of the two constraints -carbon budget and minerals stock- weighing on the economy. Using comparative dynamics, we are also able to analyze the consequences of a more stringent climate policy, of a lower investment cost in green capital and of improved technologies to invest in green capital, on investment decisions and the energy mix, hence on the time when fossil fuels stop being an energy source.

Turning to policy design, we consider the decentralized version of the economy, where the only market failure consists in the carbon budget. The policy maker can implement the optimal policy, by levying a a specific carbon tax. This policy may not be feasible if the regulator is constrained in the policy tools it can use. We consider the case where the regulator can charge a constant carbon tax, insufficiently high to ensure that the carbon budget is respected. It can use the carbon tax revenues to promote the production of renewable energy, through demand-pull subsidies, such as feed-in tariffs and feed-in premiums. We analyze the case where the latter are used, to ensure the carbon budget.

Given that the policy makers typically ignore potential scarcity of minerals embedded in green capital, we model a regulator that sets a constant carbon tax in the aim of respecting the carbon budget, and does so as if minerals were abundant. In doing so, we can see by how much cumulative emissions overshoot the carbon budget and the expected date of fossil phase-out is mistaken.

In addition, the model is extended to allow for recycling, which is costly but allows the critical resource to be exhausted less quickly. Again, we solve the program by considering different potential successions of phases. For instance, recycling may start before minerals are exhausted and backtop technology appears, but after carbon budget is reached. Alternatively, it may well be the case that recycling starts first, followed by carbon budget saturation and minerals exhaustion. Do note however, that if the expensive backstop is used before any recycling has started, it will then never be optimal to recycle solar panels. Comparative dynamics are conducted to analyze the consequences of a reduction in the cost of recycling, in terms of green capital accumulation and optimal time to abandon fossil fuel based electricity generation. We analyze the effects of a public policy in favor of recycling on the dynamics of the energy transition.

Section 2 presents the model of optimal energy transition under a scarcity constraint on minerals. Section 3 considers the decentralized equilibrium and analyzes first best, second best and myopic regulation. In section 4 the model is extended to consider costly endogenous recycling of the minerals.

2 Optimal energy transition under minerals' scarcity

In this version of the model, investment in green capital, Y, requires the use of minerals or CRMs, m, that are exhaustible. There exists a CRMs-free backstop technology that is very costly, hence assumed not be used at the beginning of the program. The benevolent social planner's problem is:

$$\max \int_0^\infty e^{-\rho t}[u(e(t)) - C(I(t)) - \nu b(t)]dt \tag{1}$$

$$e(t) = x(t) + \phi Y(t) \tag{2}$$

$$I(t) = \chi m(t) + b(t) \tag{3}$$

$$\dot{X}(t) = \varepsilon x(t) \tag{4}$$

$$\dot{Y}(t) = I(t) - \delta Y(t) \tag{5}$$

$$\dot{M}(t) = -m(t) \tag{6}$$

$$X(t) \leq \overline{X}, \quad x(t) \geq 0, \quad M(t) \geq 0, \quad m(t) \geq 0, \quad b(t) \geq 0$$

$$X(0) = X_0 \geq 0, \quad Y(0) = Y_0 \geq 0 \text{ and } M(0) = M_0 \geq 0 \text{ given}$$

$e(t)$ is the electricity consumption. $u(e(t))$ is the instantaneous utility function, assumed increasing and concave. The social discount rate is ρ. $x(t)$ is the fossil resource use as well as the production of electricity from these sources. $Y(t)$ is green capital. ϕ measures the efficiency of green electricity generation. Hence $\phi Y(t)$ is the consumption of electricity from renewable sources. $I(t)$ is the investment in green capital. $C(I(t))$ is the cost function for investment in green capital, assumed increasing and strictly convex to reflect adjustment costs. Investment consists in primary mineral resources, $m(t)$, converted into specialized equipment at rate χ, and in the backstop input $b(t)$, available at constant cost ν. The unit and marginal costs of production of fossil and mineral resources are assumed to be nil.Green capital depreciates at the constant rate δ. $X(t)$ is the stock of carbon accumulated into the atmosphere due to fossil fuel combustion. ε is the emission coefficient. $\bar{X}$ is the carbon budget, that is the ceiling on the atmospheric carbon concentration targeted by climate policy.

In what follows we set the scale parameters $X_0 = 0$ and $\chi = 1$.

The current value Hamiltonian associated to the social planner's program (1) is:

$$\mathcal{H}(.) = u(x + \phi Y) - C(m + b) - \nu b - \lambda \varepsilon x + \mu(m + b - \delta Y) + \zeta(-m)$$

where λ is the shadow price of the carbon, that is the carbon value, μ is the shadow price of the green capital and ζ is the shadow price of the stock of minerals.

Introducing Lagrange multipliers for the different inequality constraints allows us to write the Lagrange function as:

$$\mathcal{L}(.) = \mathcal{H}(.) + \omega_x x + \omega_m m + \omega_b b + \omega_X(\bar{X} - X) + \omega_M M$$

First order optimality conditions are:

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \iff u'(e) - \lambda\varepsilon + \omega_x = 0 \iff u'(e) = \varepsilon\lambda - \omega_x \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial m} = 0 \iff -C'(I) + \mu - \zeta + \omega_m = 0 \iff C'(I) = \mu - \zeta + \omega_m \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial b} = 0 \iff -C'(I) - \nu + \mu + \omega_b = 0 \iff C'(I) + \nu = \mu + \omega_b \tag{9}$$

$$-\frac{\partial \mathcal{L}}{\partial X} = \omega_X = -\dot{\lambda} + \rho\lambda \iff \dot{\lambda} = \rho\lambda - \omega_X \tag{10}$$

$$-\frac{\partial \mathcal{L}}{\partial Y} = -\phi u'(e) + \delta\mu = \dot{\mu} - \rho\mu \iff \dot{\mu} = (\rho + \delta)\mu - \phi u'(e) \tag{11}$$

$$-\frac{\partial \mathcal{L}}{\partial M} = -\omega_M = \dot{\zeta} - \rho\zeta \iff \dot{\zeta} = \rho\zeta - \omega_M \tag{12}$$

The solution is a trajectory through different phases, during which different sets of non-negativity constraints are binding (the Lagrangian multipliers in the system above are not nil). Abstracting from cases where resource use is intermittent, we define the intervals of time when each resource is used with the following notation

$$x(t) > 0 \ \forall t \in [T_x, T_X), \quad m(t) > 0 \ \forall t \in [T_m, T_M), \quad b(t) > 0 \ \forall t \in [T_b, T_B) \tag{13}$$

In what follows we consider only the cases when both fossils and minerals are used from the beginning of the program, i.e. $T_x = T_m = 0$, which requires specific restrictions on the set of parameters and the initial size of the stocks X_0, Y_0 and M_0. From the assumption that minerals and the backstop technology are perfect substitutes for investment in green capital (3), and that the backstop technology is initially relatively expensive, i.e. $\zeta(0) < \nu$ in (8)-(9), it follows that the backstop is used later than the minerals: $T_m \leq T_b$.

We first present the case when the carbon budget is exhausted before the exhaustion of minerals, then consider the opposite sequence.

2.1 Carbon budget binding first

In the first optimal sequence that we characterize there are three phases: during the first one both fossil and mineral resources are used, during the second one only mineral resources are used, and during the last phase only the backstop technology is used.

First phase: $x > 0$, $m > 0$, $b = 0 \Rightarrow \omega_x = \omega_m = 0$, $\omega_b > 0$. The carbon ceiling and the mineral scarcity are not binding: $\omega_X = \omega_M = 0$.

Equations (7) and (2) yield

$$u'(x(t) + \phi Y(t)) = \varepsilon\lambda(t) \tag{14}$$

Hence, given the assumption that fossil resources and green capital are perfect substitutes in the production of electricity, the marginal utility of electricity consumption is equal to the carbon price, at each date when fossil resources are used.

Equation (8) yields

$$C'(m(t)) = \mu(t) - \zeta(t) \iff C'(m(t)) + \zeta(t) = \mu(t) \tag{15}$$

The total marginal cost of using minerals to build up green capital (marginal investment cost plus scarcity rent, since the extraction cost is nil) is equal to the value of green capital.

This condition is relevant if and only if $\mu(t) > \zeta(t)$, i.e. the value of green capital is higher than the value of the mineral stock under the ground.

Moreover, according to (9), $C'(m(t)) + \nu > \mu(t)$. Then we necessarily have during this phase $\zeta(t) < \nu$: the scarcity rent of the mineral stock is smaller than the marginal cost of the backstop, which explains why the backstop is not used.

Equations (10) and (12) imply that the Hotelling rule applies to the two scarcity rents (fossil and minerals), both characterized by zero extraction costs:

$$\lambda(t) = \lambda(0)e^{\rho t} \quad \text{and} \quad \zeta(t) = \zeta(0)e^{\rho t} \tag{16}$$

Bringing together, on the one hand (11) and (14), and, on the other hand, (5), (3) and (15), the dynamics of the economy is given by

$$\forall t \in [0, T_X) \quad \begin{cases} \dot{\mu}(t) = (\rho + \delta)\mu(t) - \phi\varepsilon\lambda(t) \\ \dot{Y}(t) = C'^{-1}\left(\mu(t) - \zeta(t)\right) - \delta Y(t) \end{cases} \tag{17}$$

The first phase ends at date T_X, such that for the first time $x(t) = 0 \ \forall t \geq T_X$, and $X(T_X) = \bar{X}$. T_X is defined by (14):

$$u'(\phi Y(T_X)) = \varepsilon\lambda(T_X) \tag{18}$$

and

$$\int_0^{T_X} x(t)dt = \overline{X}/\varepsilon \tag{19}$$

Second phase: $x = 0$, $m > 0$, $b = 0 \Rightarrow \omega_x > 0$, $\omega_m = 0$, $\omega_b > 0$. Until mineral scarcity binds: $\omega_M = 0$.

We still have (15), and therefore $\mu(t) \in (\zeta(t), C'(m(t)) + \nu)$, since $\zeta(t) < \nu$. Taking it into account together with (5) and (3), as well as substituting $e(t) = \phi Y(t)$ into (11), the dynamics of the economy is given by

$$\forall t \in [T_X, T_M) \quad \begin{cases} \dot{\mu}(t) = (\rho + \delta)\mu(t) - \phi u'(\phi Y(t)) \\ \dot{Y}(t) = C'^{-1}\left(\mu(t) - \zeta(t)\right) - \delta Y(t) \end{cases} \tag{20}$$

Let us study the end of the phase. First note that $\mu(t)$ is continuous at the time of the switch form phase 2 to phase 3 as the level of the associated state variable Y is free (see Boucekkine *et al.*, 2013). In addition, equations (8) and (9) $-\zeta(t)+\omega_m = -\nu+\omega_b$ with ζ_t increasing and ν constant. Hence it is only possible to have $-\zeta(t) = -\nu$ at one point in time, such that $T_m = T_b$ at the end of the phase.
The rest of the programme, either ω_M or ω_b is different from zero implying that there is no phase of simultaneous use of minerals and the backstop to invest in green capital.

At the end of the second phase, these two costs are equal, the mineral stock is exhausted and minerals are replaced by the backstop. This occurs at date T_M defined by $\zeta(T_M) = \nu$, when

$$C'(I(T_M)) + \nu = \mu(T_M) \quad \text{with} \quad I(T_M) = m(T_M^-) = b(T_M^+) \tag{21}$$

and

$$\int_0^{T_M} m(t)dt = M_0 \tag{22}$$

Third phase: $x = m = 0$, $b > 0 \Rightarrow \omega_x > 0$, $\omega_m > 0$, $\omega_b = 0$.

Green investment in (5) consists exclusively of the backstop technology according to (9) $b(t) = C'^{-1}(\mu(t) - \nu)$. Again using $e(t) = \phi Y(t)$ into (11), the dynamics of the economy is given by

$$\forall t \in [T_M, T_B) \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \phi u'(\phi Y(t)) \\ \dot{Y}(t) = C'^{-1}(\mu(t) - \nu) - \delta Y(t) \end{cases} \tag{23}$$

This system converges to a steady state (μ^*, Y^*), defined by:

$$\begin{cases} (\rho+\delta)\mu^* = \phi u'(\phi Y^*) \\ C'^{-1}(\mu^* - \nu) = \delta Y^* \end{cases} \tag{24}$$

hence $T_B = \infty$. The steady state is saddle path.

Sub-case with quadratic costs and logarithmic utility.

Consider

$$u(e(t)) \equiv \gamma \ln e(t) \tag{25}$$

$$C(I) \equiv c_1 I + \frac{c_2}{2} I^2 \quad \Rightarrow \quad C'(I) = c_1 + c_2 I, \quad C'(0) = c_1 \tag{26}$$

Then, the optimal trajectory is given by

$$\forall t \in [0, T_X) \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \phi\varepsilon\lambda(t) \\ \dot{Y}(t) = \frac{1}{c_2}(\mu(t) - \zeta(t) - c_1) - \delta Y(t) \end{cases} \tag{27}$$

$$\forall t \in [T_X, T_M) \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \gamma/Y(t) \\ \dot{Y}(t) = \frac{1}{c_2}(\mu(t) - \zeta(t) - c_1) - \delta Y(t) \end{cases} \tag{28}$$

$$\forall t \geq T_M \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \gamma/Y(t) \\ \dot{Y}(t) = \frac{1}{c_2}(\mu(t) - \nu(t) - c_1) - \delta Y(t) \end{cases} \tag{29}$$

and approaches the steady state at which

$$\mu^* = \frac{\nu + c_1}{2}\left(1 + \sqrt{1 + 4\frac{c_2\delta\gamma}{(\rho+\delta)(\nu+c_1)^2}}\right) \quad , \quad Y^* = \frac{\mu^* - \nu - c_1}{c_2\delta} \tag{30}$$

For this sequence to be optimal, the following restrictions must hold

$$\frac{\gamma}{\varepsilon\phi Y^*} < \lambda(0) < \frac{\gamma}{\varepsilon\phi Y_0} \quad \Leftrightarrow \quad x(0) > 0,\ T_X > 0 \text{ finite} \tag{31}$$

$$\zeta(0) < \nu < \infty \quad \Leftrightarrow \quad b(0) = 0, \quad \zeta(0) < \mu(0) - c_1 \quad \Leftrightarrow \quad m(0) > 0,\ T_M > 0 \text{ finite} \tag{32}$$

These conditions restrict the set of exogenous values of the stocks $\bar{X}, M_0, Y_0$.

2.2 Minerals scarcity binding first

Let us now turn to the case when, along the optimal trajectory, minerals are exhausted before the carbon budget: $T_M < T_X$. We still restrict our analysis to the case when both fossils and minerals are used from the beginning of the program, i.e. $T_x = T_m = 0$. Here too the optimal sequence is characterized by three phases: during the first one both fossil and mineral resources are used, during the second one fossil resources and the backstop technology are used, and during the last phase only the backstop technology is used.

First phase: $x > 0$, $m > 0$, $b = 0 \Rightarrow \omega_x = \omega_m = 0$, $\omega_b > 0$.

The economy satisfies equations (14)-(17), but for substituting T_M for T_X in the latter. This phase comes to an end at T_M, when mineral resources are exhausted, and (21)-(22) hold.

Second phase: $x > 0$, $m = 0$, $b > 0 \Rightarrow \omega_x = \omega_b = 0$, $\omega_m > 0$.

As for the first phase, we take into account in the law of motion of the value of green capital (11) the fact that the marginal utility of electricity consumption equates the value of carbon, according to (14). Moreover, green investment in (5) consists exclusively of the backstop technology according to (9) $b(t) = C'^{-1}(\mu(t) - \nu)$. Hence the dynamics of the economy is given by

$$\forall t \in [T_M, T_X) \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \phi\varepsilon\lambda(t) \\ \dot{Y}(t) = C'^{-1}\left(\mu(t) - \nu\right) - \delta Y(t) \end{cases} \tag{33}$$

Date T_X, by which this phase ends, satisfies conditions (18)-(19).

Third phase: $x = m = 0$, $b > 0 \Rightarrow \omega_x > 0$, $\omega_m > 0$, $\omega_b = 0$.

Once fossil and mineral resources are exhausted, the optimal dynamics of the economy is identical to the one described by in the previous case, i.e. (23), and leads toward the same steady state (24).

Sub-case with quadratic costs and logarithmic utility.

Under the specification (25)-(26), the trajectory is described by (29)-(32), (27) with T_M replacing T_X, and (28) replaced by

$$\forall t \in [T_M, T_X) \quad \begin{cases} \dot{\mu}(t) = (\rho + \delta)\mu(t) - \phi\varepsilon\lambda(t) \\ \dot{Y}(t) = \frac{1}{c_2}(\mu(t) - \zeta(t) - c_1) - \delta Y(t) \end{cases} \tag{34}$$

3 Public policy for the energy transition under mineral scarcity

In this section we consider a decentralized economy, corresponding to the technology and preferences assumed in the previous section, to analyze the role public policy. We proceed in four steps. First, we present the economy and characterize the equilibrium trajectory for a specific set of public intervention. Second, we describe the policy that allows the equilibrium trajectory to coincide with the optimal one. Third, we study a second best policy that the regulator can implement, under the assumption that its policy tools are constrained in a specific way, reminiscent of widespread policies implemented in the world to stimulate the production of electricity from renewable sources of energy. Fourth, we consider a myopic regulator who sets, once and for all, these policy tools without taking into account the scarcity of minerals, and show how the equilibrium trajectory of the economy differs from the one that the regulator was targeting. This last step gives substance to the main message of the article: it may be misleading to ignore the scarcity of minerals that are critical to investment in specific infrastructure and capital for the energy transition.

3.1 The decentralized economy

Consider an economy with a representative electricity consumer, with utilities producing electricity from either fossil resources or a specific capital, dubbed green capital, that they own, with owners of mineral resources and owners of fossil resources. Investment in green capital by utilities mobilizes either mineral resources or the backstop technology, available in-house at a constant cost, ν. All markets are perfectly competitive, including a capital market where the rate of interest is denoted r. We adopt a partial equilibrium approach.[3]

The regulator has several policy tools: a tax on carbon emissions, τ, a subsidy on investment in green capital, s, a feed-in-premium (FIP) for electricity produced from green capital, σ, a tax on electricity consumption, η, and finally a lump sum transfer to the household, $\mathcal{T}$.

The representative consumer is characterized by the preference structure outlined in (1). She faces a given purchaser price of electricity given by the sum of the tax on electricity consumption and the seller price of electricity, P_e. Hence, the program of the electricity consumer is $\forall t \geq 0$

$$\max_{e_t} u(e(t)) - (P_e(t) + \eta(t))\, e(t) + \mathcal{T}(t) \tag{35}$$

The first order condition of this problem gives the electricity demand schedule

$$u'(e(t)) = P_e(t) + \eta(t) \tag{36}$$

[3]We assume a constant interest rate, though the rate of growth of electricity consumption is not constant.

The electricity is supplied by the representative power utility. There is perfect competition on the electricity market and the electricity price P_e is taken as given by the representive firm. This firm seeks to maximize the present value of its profits, taking into account that building up its green capital to produce renewable energy requires specific inputs, either minerals or the backstop in-house technology, and implies adjustment costs. It therefore solves an intertemporal program, based on the expected evolution of the electricity price P_e, the prices of the inputs, fossil resources P_x and minerals P_m, as well as policy tools. We assume that subsidy, the FIP and the carbon tax grow at constant, possibly nil, rates, g_s, g_σ and g_τ respectively.

The program of the power producer is:

$$\begin{array}{ll}\max\limits_{x(t),m(t),b(t)} & \int_0^\infty [P_e(t)x(t) + \left(P_e(t) + \sigma_0 e^{g_\sigma t}\right)\phi Y(t) - \left(P_x(t) + \varepsilon\tau_0 e^{g_\tau t}\right)x(t) \\ & \quad -C(I(t)) - \left(P_m(t) - s_0 e^{g_s t}\right)m(t) - \left(\nu - s_0 e^{g_s t}\right)b(t)]e^{-rt}dt \\ s.t. & \dot{Y}(t) = I(t) - \delta Y(t) \\ & I(t) = m(t) + b(t) \\ & x(t) \geq 0,\ m(t) \geq 0,\ b(t) \geq 0 \end{array}$$

Denoting by μ_d the value of green capital for the power company, the associated Lagrangian is:

$$\begin{aligned}\mathcal{L} = P_e(t)x(t) + \left(P_e(t) + \sigma_0 e^{g_\sigma t}\right)\phi Y(t) - \left(P_x(t) + \varepsilon\tau_0 e^{g_\tau t}\right)x(t) - C(m(t)+b(t)) - \left(P_m(t) - s_0 e^{g_s t}\right)m(t) \\ - \left(\nu - s_0 e^{g_s t}\right)b(t) + \mu_d(t)(m(t) + b(t) - \delta Y(t)) + \omega_x x(t) + \omega_m m(t) + \omega_b b(t)\end{aligned}$$

From which the first order conditions and Euler equation are the following

$$\frac{\partial \mathcal{L}}{\partial x(t)} = P_e(t) - P_x(t) - \varepsilon\tau_0 e^{g_\tau t} + \omega_x = 0 \tag{37}$$

$$\frac{\partial \mathcal{L}}{\partial m(t)} = -P_m(t) + s_0 e^{g_s t} - C'(I(t)) + \mu_d(t) + \omega_m = 0 \tag{38}$$

$$\frac{\partial \mathcal{L}}{\partial b(t)} = -\nu + s_0 e^{g_s t} - C'(I(t)) + \mu_d(t) + \omega_b = 0 \tag{39}$$

$$\dot{\mu_d}(t) = (r+\delta)\,\mu_d(t) - \left(P_e(t) + \sigma_0 e^{g_\sigma t}\right)\phi \tag{40}$$

On the mineral resources market, suppliers do not have any production cost, but have a limited stock of resource to sell. They are therefore willing to sell all the stock at a pace that depends on the dynamics of the selling price, hence the scarcity rent. Consider a unit mass of mineral resources producers. The program of the representative firm $i \in [0,1]$, owning a stock M_{i0}, is:

$$\begin{cases} \max\limits_{m_i(t)} \int_0^\infty e^{-rt} P_m(t) m_i(t) dt \\ s.t. \quad \int_0^\infty m_i(t)dt = M_{i0} \end{cases} \tag{41}$$

Denoting by ζ_d the value of the resource stock, the associated Hamiltonian is:

$$\mathcal{H}_i = P_m(t)m_i(t) - \zeta_d(t)m_i(t) \tag{42}$$

The first order and Euler conditions are

$$P_m(t) = \zeta_d(t) \tag{43}$$

$$\frac{\dot{\zeta}_d(t)}{\zeta_d(t)} = r \tag{44}$$

implying the Hotelling rule

$$\frac{\dot{P}_m(t)}{P_m(t)} = r \tag{45}$$

In addition, at equilibrium upon exhaustion of the mineral resource, green capital investment starts relying on the backstop technology, and no minerals are left unexploited. Hence at date T_M

$$P_m(T_M) = \nu \tag{46}$$

$$\int_0^{T_M} \left(\int_0^1 m_i(t) di \right) dt = M_0 \tag{47}$$

which determine the value of $P_m(0) = \zeta_d(0)$.

On the market for fossil resources, the scarcity of the natural resource stock is not binding at equilibrium, since the environmental problem is more stringent than resource scarcity. In effect, firms act as if their resources were abundant, so that, rather than solving an intertemporal optimization problem, they maximize their current profit at each date. Given that the production costs are assumed nil and that the market is perfectly competitive, the supply schedule of fossil resources is perfectly elastic at seller price $\forall t \geq 0$

$$P_x(t) = 0 \tag{48}$$

The quantity of fossil resources sold is determined by the demand originating from power suppliers.

One can consider different objectives to set public policy. For instance, one can assume that the government maximizes total surplus. Alternatively, that it minimizes the social cost of avoiding overshooting an exogenous carbon budget, $\bar{X}$. In line with our approach in Section 2, we assume the latter.

The public budget constraint is:

$$\tau(t)\varepsilon x(t) + \eta(t)e(t) = s(t)\left(m(t) + b(t)\right) + \sigma(t)\phi Y(t) + \mathcal{T}(t) \tag{49}$$

and, by assumption, $\tau(t) = \tau_0 e^{g_\tau t}$, $s(t) = s_0 e^{g_s t}$ and $\sigma(t) = \sigma_0 e^{g_\sigma t}$.

3.2 Optimal policy

If $r = \rho$, and the public policy consists of a carbon tax equal to the optimal value of carbon $\tau(t) = \lambda(t)$ (i.e. $g_\tau = \rho$ and $\tau_0 = \lambda(0)$ determined in Section 2), with the tax revenue transferred lump-sum at each date to the consumer (i.e. $s_0 = \sigma_0 = \eta(t) = 0$), the economy evolves along the optimal trajectory.

To see this, let us consider the case when the carbon budget binds first. During the first phase, we have $x(t) > 0$, $m(t) > 0$, and $b(t) = 0$, thus $\omega_x = 0$. Taking this into account together with (48) in the power supplier condition (37), we find that the electricity seller price is determined by the carbon tax:

$$P_e(t) = \varepsilon\tau_0 e^{g_\tau t} \tag{50}$$

Using this together with $\eta(t) = 0$ in the demand function for electricity (36), we see that at equilibrium electricity consumption is entirely driven by the carbon tax

$$e(t) = u'^{-1}\left(\varepsilon\tau_0 e^{g_\tau t}\right) \tag{51}$$

Taking into account that $I(t) = m(t)$ and $\omega_m = 0$, mineral use results of (38), and is

$$m(t) = C'^{-1}\left(\mu_d(t) - P_m(t)\right) \tag{52}$$

Using this into the law of motion of green capital, and (50) into (40) with $\sigma_0 = 0$, the equilibrium dynamics of the economy is

$$\forall t \in [0, T_X) \quad \begin{cases} \dot{\mu}(t) = (r+\delta)\mu(t) - \phi\varepsilon\tau_0 e^{g_\tau t} \\ \dot{Y}(t) = C'^{-1}\left(\mu(t) - P_m(t)\right) - \delta Y(t) \end{cases} \tag{53}$$

This phase ends at date T_X, such that $x(t) = 0\ \forall t \geq T_X$, so that $e(T_X) = \phi Y(T_X)$ in (37):

$$u'(\phi Y(T_X)) = \varepsilon\tau_0 e^{rT_X} \tag{54}$$

where $Y(T_X) = Y_0 + \int_0^{T_X}\dot{Y}(t)dt$ from (53). Since the production of electricity from renewable sources depends entirely on the dynamics of the stock of green capital during this phase, the use of fossil resources is determined as the residual to ensure that the electricity supply covers demand (51)

$$x(t) = u'^{-1}\left(\varepsilon\tau_0 e^{g_\tau t}\right) - \phi Y(t) \tag{55}$$

In order to ensure that the carbon ceiling is attained, but not overshot, the choice of τ_0 and g_τ should be such that

$$\int_0^{T_X} x(t)dt = \overline{X}/\varepsilon \tag{56}$$

Suppose that the regulator knows the optimal trajectory studied in Section 2.1, and in particular the initial values $\mu(0)$, $\zeta(0)$ and $\lambda(0)$. During this first phase, if $r = \rho$, $\mu_d(0) = \mu(0)$ and $P_m(0) = \zeta(0)$, then by levying the carbon tax with $\tau_0 = \lambda(0)$ and $g_\tau = \rho$ the regulator allows the equilibrium to follow the optimal trajectory, since (53) coincides with (17), and the date of fossil phase-out T_X satisfies the same conditions (54)-(56) as (18)-(19) with $x(t)$ resulting of (16) in (14).

During the second and third phases, $x(t) = 0$ so that $e(t) = \phi Y(t)$ in (36) can be used to determine the dynamics of μ_d as function of Y from (40), to get

$$\forall t \geq T_X \qquad \dot{\mu_d}(t) = (r+\delta)\,\mu_d(t) - \phi u'(\phi Y(t)) \tag{57}$$

Minerals are exhausted at date T_M, with their price reaching smoothly the chuck off price ν. If $r = \rho$ then, comparing (45)-(47) to (16) and (22) , we see that $P_m(t) = \zeta(t)$ at least for dates close to T_M. Equation (22) ensures that $P_m(0) = \zeta_d(0)$ hence the two paths perfectly coincide and the equilibrium time of exhaustion is the optimal one. Date T_M satisfies (46)-(47) with extraction following (52), which can be compared to conditions (21)-(22) and (15) determining the optimal date, to conclude that the equilibrium date T_M is optimal since $\mu_d(t) = \mu(t)$ is ensured by the equality of the steady-state values of Y.

During the third phase, investment in green capital relies exclusively on the backstop technology. Hence, $I(t) = b(t)$ and $\omega_b = 0$ in (39), together with $\sigma_0 = 0$, hand the gross investment. Using this into the law of motion of green capital, we get

$$\forall t \geq T_M \qquad \dot{Y}(t) = C'^{-1}\left(\mu_d(t) - \nu\right) - \delta Y(t) \tag{58}$$

Since the value of green capital evolves according to (57), the dynamics of the economy at equilibrium coincide with the optimal one over the last phase, as the $\mu_d(T_M)$ and $Y(T_M)$ are indeed optimal. The economy converges to the optimal steady state, given by (24).

We conclude that, if market agents refer the social discount rate (i.e. if $r = \rho$), the regulator levies the optimal carbon tax to correct for the only externality in the economy: the cap on carbon emissions that, in the absence of environmental regulation, market operators do not take into account. The optimal carbon tax consists of a tax per unit of carbon emissions that grows at the social rate of discount from the specific level $\tau_0 = \lambda(0)$, where $\lambda(0)$ is the solution of the problem in Section 2.1.

Mutatis mutandis the analysis holds for the case when mineral scarcity binds first.

Sub-case with quadratic costs and logarithmic utility.

In the aftermath of this section we restrict the analysis to this sub-case. We have that (55) hands

$$x(t) = \begin{cases} \frac{\gamma}{\varepsilon\tau_0} e^{-g_\tau t} - \phi Y(t) & t < T_X \\ 0 & t \geq T_X \end{cases} \tag{59}$$

(52) gives

$$m(t) = \begin{cases} \frac{1}{c_2}\left(\mu_d(t) - P_m(0)e^{rt} - c_1\right) & t < T_M \\ 0 & t \geq T_M \end{cases} \tag{60}$$

while

$$b(t) = \begin{cases} 0 & t < T_M \\ \frac{1}{c_2}\left(\mu_d(t) - \nu - c_1\right) & t \geq T_M \end{cases} \tag{61}$$

The dynamics of green capital is therefore

$$\dot{Y}(t) = \begin{cases} \frac{1}{c_2}\left(\mu_d(t) - P_m(0)e^{rt} - c_1\right) - \delta Y(t) & t < T_M \\ \frac{1}{c_2}\left(\mu_d(t) - \nu - c_1\right) - \delta Y(t) & t \geq T_M \end{cases} \tag{62}$$

and the one of its value is

$$\dot{\mu}_d(t) = \begin{cases} (r+\delta)\mu_d(t) - \varepsilon\tau_0 e^{g_\tau t} & t < T_X \\ (r+\delta)\mu_d(t) - \gamma/Y(t) & t \geq T_X \end{cases} \tag{63}$$

The steady state is given by (30), with $\mu(t)$ replaced by $\mu_d(t)$ and ρ by r.

3.3 Constant carbon tax and FIP with strict balanced budget

We now introduce constraints on the set of policy tools available to the regulator. In several countries the announcement of climate change mitigation policies, including a increasing carbon tax, have received strong political opposition. This opposition is weaker when the revenue from the tax finances renewable energy production.

In order to represent real world policy making, we assume in this section that the government can commit to levy a constant carbon tax, but not an increasing one and that the proceed of this tax is used to subsidize investment in renewable energy production. Indeed, in most countries climate mitigation policies do not rely exclusively on pricing carbon emissions.They typically include interventions in the power market with demand-pull instruments targeted at electricity from low-carbon energy sources. Several countries have implemented or currently implement subsidies to power production from specific sources, often through feed-in-tariffs or feed-in-premiums (FIP) for investments in power plants.

If for some reason, subsiding renewables is politically more acceptable than taxing carbon emissions, governments might combine these tools to target specific objectives, such as limiting emissions within a carbon budget, or ensuring fossil phase out by some date. In this section we assume that the government objective is to respect an exogenous carbon budget (in order to drive comparisons with the optimal case).

In this section, we therefore restrict the scope of policy tools to a constant carbon tax, τ, and a (variable) feed-in-premium on the seller price of electricity, $\sigma(t)$, when the latter is produced from green capital. Moreover, we assume that the regulator is constrained to choose a combination of these two instruments, such that the budget is balanced at each date. Hence the budget balance (49) simplifies to

$$\tau\varepsilon x(t) = \sigma(t)\phi Y(t) \tag{64}$$

Substituting for $x(t) = e(t) - \phi Y(t)$ from the definition of electricity consumption, and rearranging, we see that the FIP depends on the carbon tax and on the electricity mix:

$$\sigma(t) = \left(\frac{e(t)}{\phi Y(t)} - 1\right)\tau\varepsilon \in \left[0, \tau\frac{\varepsilon x(0)}{\phi Y_0}\right] \tag{65}$$

It falls as the share of renewables in the electricity mix increases.

With $\eta(t) = 0$, the consumer demand function is then $\gamma/e(t) = P_e(t)$.

During the first phase, the choice of fossil resource use by power producer (37) is now

$$P_e(t) = \tau\varepsilon \tag{66}$$

Combining this with the demand for electricity (36), we note that electricity consumption is constant as long a fossil resources are used for power production:

$$e(t) = \gamma/(\tau\varepsilon), \qquad t \leq T_X \tag{67}$$

and using the definition of $e(t)$

$$x(t) = \gamma/(\tau\varepsilon) - \phi Y(t), \qquad t \leq T_X \tag{68}$$

Hence, as the stock of green capital increases, fossil resource use decreases from $x(0) = \gamma/(\tau\varepsilon) - \phi Y_0$ (notice that $\tau < \gamma/\left(\varepsilon\phi Y_0\right) \Leftrightarrow x(0) > 0$), down to $x(T_X) = 0$ when $Y(T_X) = \gamma/(\tau\varepsilon\phi)$ (notice that $\tau > \gamma/\left(\varepsilon\phi Y^*\right) \Leftrightarrow Y(T_X) < Y^*$).

Using (67) and (68) into (65) we can express the FIP as

$$\sigma(t) = \begin{cases} \gamma/(\phi Y(t)) - \tau\varepsilon & t < T_X \\ 0 & t \geq 0 \end{cases} \tag{69}$$

For the electricity producer the value of green capital evolves according to

$$\dot{\mu}_d(t) = (r + \delta)\,\mu(t)_d - (P_e(t) + \sigma(t))\,\phi \tag{70}$$

(66) and (69) imply that during the first phase $(P_e(t) + \sigma(t))\phi = \gamma/Y(t)$. During the following phases, $e(t) = \phi Y(t)$, which in the demand function together with $\sigma(t) = 0$ with (65) imply again $(P_e(t) + \sigma(t))\phi = \gamma/Y(t)$. Therefore[4]

$$\forall t \geq 0 \qquad \dot{\mu}_d(t) = (r + \delta)\,\mu(t)_d - \gamma/Y(t) \tag{71}$$

As a consequence, the equilibrium dynamics of green capital accumulation is characterized by only two phases, where the value of green capital follows (71) while the stock of green capital evolves according to (62). The economy convergences to the steady state given by (30), with $\mu(t)$ replaced by $\mu_d(t)$ and ρ by r.

ϕ	ρ	δ	ε	γ	ν	c_1	c_2	Y_0	M_0	$\overline{X}$
0.9	0.03	0.01	0.25	1.8	5	0.15	20	1	6	203.28

Table 1: Parameters for the baseline simulation.

Figure 1 plots the trajectories of policy tools and of endogenous variables to compare the optimum with the constrained policy. The parameters' values used for this simulation are reported in Table 1. The system of ordinary differential equations is solved using the backward shooting from $\{\mu^*, Y^*\}$ until $Y(0) = Y_0$ (see Brunner and Strulik, 2002). The guess for the initial value of minerals, $\zeta(0)$ or $P_m(0)$, is adjusted by trial and error until cumulative mineral use equals M_0. In the case of the optimal policy, we have normalized the initial value of the carbon tax $\tau(0) = 1$, and set $g_\tau = \rho$, to compute the cumulative use of fossil resources, then used this value to define $\overline{X}$ in the simulations below. When considering the constrained policy as in this sub-section, we modified the guess on the value of the constant carbon tax, until cumulative use of fossil resources attained $\overline{X}$.

The trajectories prevailing for the optimal policy are compared in columns two and three of

[4] The fact that the value of green capital is unaffected by the carbon tax and the FIP depends crucially on the assumption of a logarithmic utility function. As shown in Appendix A.1, that is not the case for an iso-elastic utility function and elasticity of intertemporal substitution different from unity.

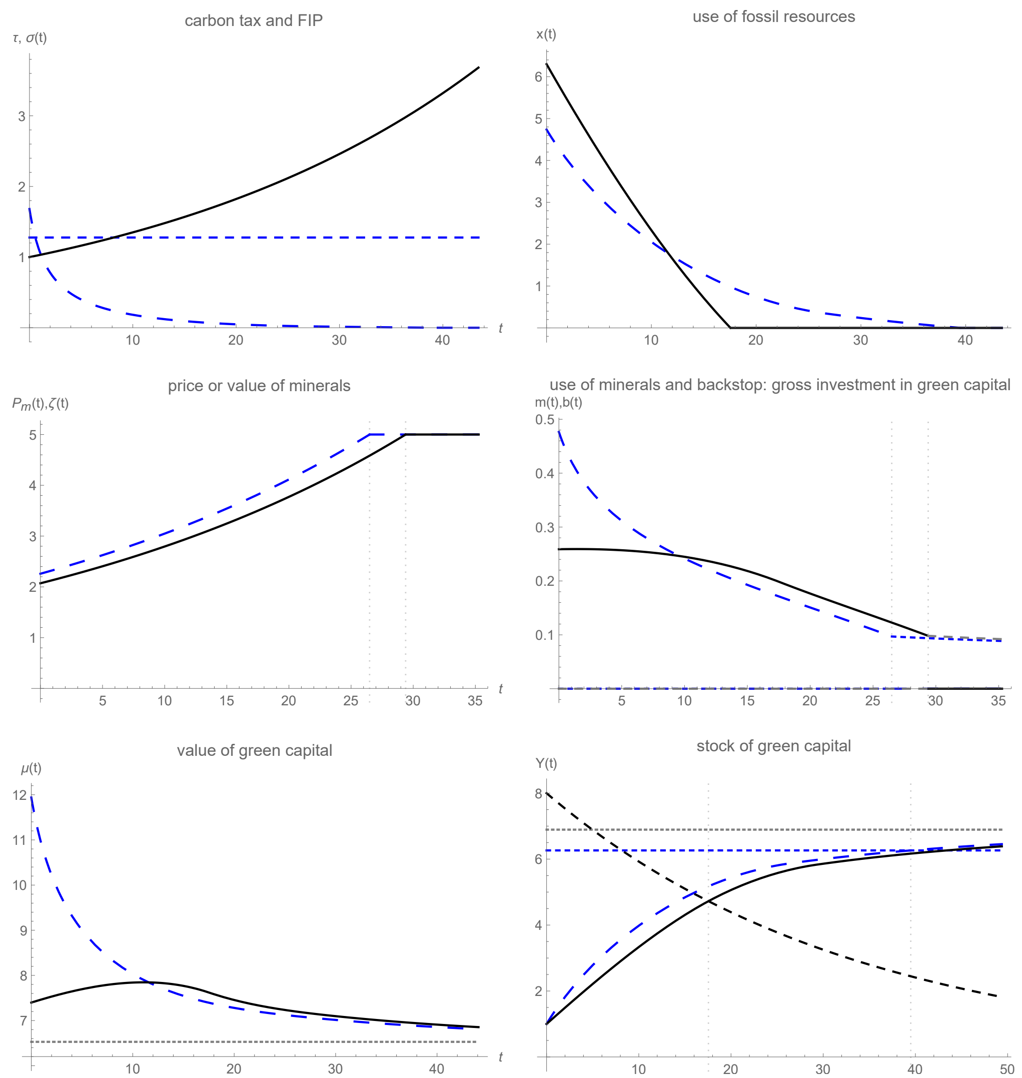

Figure 1: Optimal versus constrained policy. (Dashed blue lines for the constrained case).

Table 2 and in Figure 1. The constraint on policy tools causes a welfare loss, which is equivalent to a permanent reduction of the electricity consumption at -8.37% below its optimal path.

We conclude that the tax level chosen within this constrained policy affects only the use of fossil resources and the date of fossil phase-out. [5] Recall that in the case under analysis, the regulator effectively manages only one instrument, say the level of the carbon tax. Cumulative carbon emissions, as well as the date of fossil phase-out, decrease monotonically with the level of the carbon tax. If the objective of the regulator is expressed in terms of respecting a carbon budget, or if instead it is to phase-out fossil resources by a specific exogenous date, the policy can be designed to attain the objective. However, the goal will not be reached at the minimum social cost. Moreover, if the policy target is a couple, such as exhausting the carbon budget by a specific date, then the regulator is in general unable to implement it with the policy instruments considered in this subsection. In particular, it is impossible to implement the optimal trajectory of the economy.

[5]In particular, T_M is independent from the the tax level implying that both cases where $T_M < T_X$ and $T_X < T_M$ can appear.

While mineral resources use is independent of the policy, the opposite is not true: if mineral resources are relatively abundant, investment in green capital proceeds faster, pushing fossils out of the market. Hence, abstracting from mineral resources scarcity, will most likely be misleading for the policy design by the regulator. We next consider this issue.

	optimal policy	constrained policy	myopic forecast	myopic regulation
Welfare	39.8	37.1	44.8	37.7†
Initial carbon tax, $\tau(0)$	1	1.277	1.248	1.248
Carbon tax growth rate, g_τ	0.03	0	0	0
Initial FIP, $\sigma(0)$	0	1.681	1.737	1.688
Steady state green capital, Y^*	6.89	6.89	10.14	6.89
Fossil phase-out date, T_X	17.55	39.47	27.06	46.65
Minerals exhaustion date, T_M	29.39	26.49	-	26.49
Initial price of minerals, $P_m(0)$	2.07	2.26	2.26	2.26
Initial value of green capital, $\mu_d(0)$	7.397	11.939	11.577	11.939

†: Carbon emissions overshoot the carbon budget at no social cost.

Table 2: Comparative dynamics.

3.4 Myopic regulation

Suppose that the regulator is not aware that minerals used for investment in green capital are scarce, and that instead it believes that the supply of minerals is perfectly elastic and constant at an exogenous price $\hat{P}_m$. In the aim of attaining a given carbon budget without overshooting it, the regulator sets the carbon tax and spends the tax revenue to finance the FIP on electricity production from green capital.

The misunderstanding of the role of the global scarcity of mineral supply concerns the regulator exclusively. As a first step, however, in order to determine the chosen policy, we model the economy as the regulator does: scarcity of minerals is not taken into account by any of the agents (electricity producers and consumers, mineral and fossil resource suppliers). Doing so, we can determine the constant carbon tax to which the regulator commits from the start, as function of the initial observed price of minerals $\hat{P}_m = P_m(0)$. In a second step, we consider how this policy affects the agents in the economy, when they are aware of mineral resource scarcity, by injecting this level of the constant carbon tax in the solution analyzed in the previous subsection. In fact, the equilibrium price of minerals obtained at this stage should be consistent with the one considered by the myopic regulator $\hat{P}_m = P_m(0)$.

The main difference with respect to the previous analysis is that the problem of the mineral producer is not intertemporal any longer. We have assumed away any cost of mineral production, other that the scarcity rent. In order to make sense of the misperception of the regulator, we now assume that the latter interprets the initial observed scarcity rent, $P_m(0)$, as the constant marginal production cost, which it considers exogenous. We denote such perceived marginal and unit production cost by $\underline{\zeta}$. Hence, the regulator believes that at each date the mineral resources producer $i \in [0, 1]$ solve:

$$\max_{m_i} P_m(t)m_i(t) - \underline{\zeta} m_i(t) \tag{72}$$

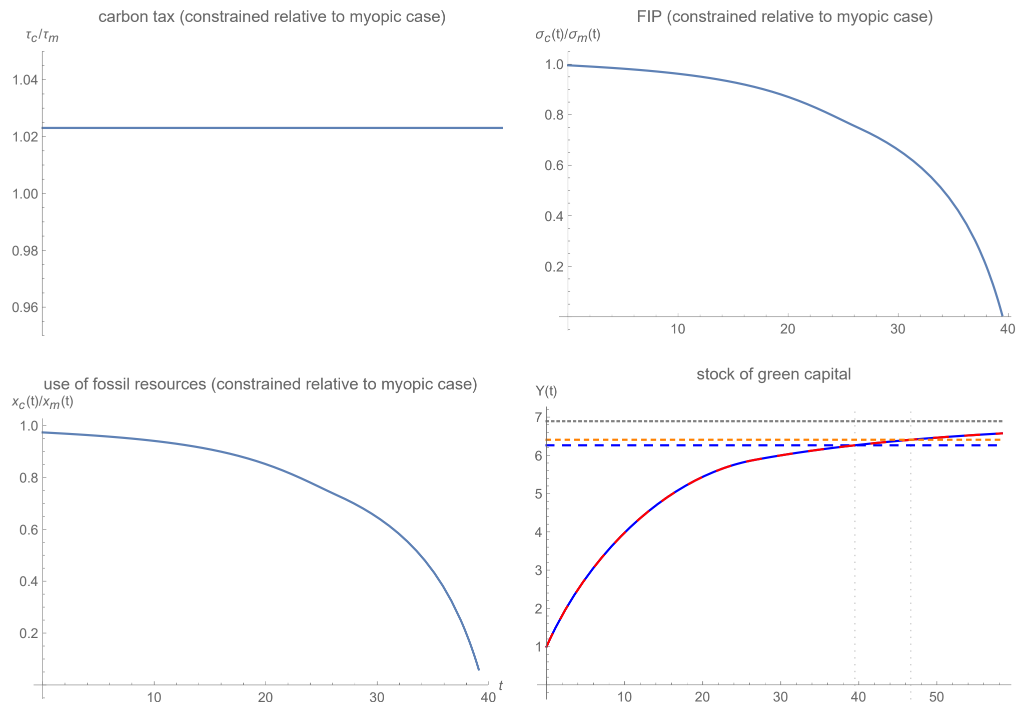

Figure 2: Constrained policy versus myopic regulation. (Dashed red lines for the myopic case).

Hence the inverse supply function of minerals is:

$$P_m(t) = \underline{\zeta} \equiv \hat{P}_m \qquad \forall m(t) \in (0, \infty) \tag{73}$$

The regulator observes that initially, minerals are used to invest in green capital. Knowing (38)-((38)), it deduces that $\underline{\zeta} < \nu$. The regulator does not imagine that the electricity producer will ever want to use the backstop technology. As a consequence, the regulator models the law of motion of the stock of green capital as follows

$$\forall t \geq 0 \qquad \dot{Y}(t) = \frac{1}{c_2}\left(\mu_d(t) - \hat{P}_m - c_1\right) - \delta Y(t) \tag{74}$$

For the rest, the analysis of the economic dynamics and of the fossil phase-out is the same as in the previous subsection. In particular, taking into account how the FIP evolves according to (69), the regulator is aware that the value of green capital for the electricity producer follows (71). The system of ordinary differential equations that it uses to model the economic dynamics is one and the same, given by (71) and (74).

In this case, the regulator hopes that the economy converges toward a different steady state:

$$\hat{\mu}^* = \frac{\hat{P}_m + c_1}{2}\left(1 + \sqrt{1 + 4\frac{c_2\delta\gamma}{(r+\delta)(\hat{P}_m + c_1)^2}}\right) \quad , \quad \hat{Y}^* = \frac{\hat{\mu}_d^* - \hat{P}_m - c_1}{c_2\delta} \tag{75}$$

with a larger stock of green capital, $\hat{Y}^* > Y^*$, and a lower value, $\hat{\mu}^* < \mu^*$, than they will actually be.

To analyze the consequence of myopic regulation, we compare it to the trajectories prevailing under constrained regulation, for parameters presented in Table 1. The myopic regulator targets the carbon budget $\overline{X} = 203.28$. It considers that the price of minerals will always be $\hat{P}_m = P_m(0) = 2.07$. It chooses the level of the constant carbon tax $\hat{\tau} = 1.2485$ that allows to meet the target $\overline{X}$, if the economy was correctly described by the model of this subsection (i.e. characterized by an infinitely elastic supply of minerals at price $\hat{P}_m$). Then the regulator commits to charge the carbon tax $\hat{\tau}$, and resources owners, power producers and electricity consumers make their choices at every period. The resulting equilibrium differs from the one planned by the regulator, aside from the initial price of minerals which is indeed equal to the one used by the regulator to model the impact of its policy.

We find that the trajectory of mineral use, and of green capital are unchanged, but that fossil resource use is affected by myopia (see Table 2). As shown in Figure 2, $x(t)$ is always larger under myopic regulation. This results in cumulative carbon emissions overshooting by 10.97% the target, reaching $X(T_X) = 225.56$. This comes from the fact that ignoring minerals' exhaustibility boils down to under-estimating their real cost, hence that of the energy transition. As a result, it leads to undersizing environmental policy. Other policy characteristics are not attained: the stock of green capital at steady state is $Y^* = 6.89$ instead of the expected $\hat{Y}^* = 10.14$; and the fossil phase-out date is $T_X = 46.65$ as compared to the myopic policy forecast of 27.06. The latter is even farther than the one prevailing under constrained regulation (39.5), because the constant carbon tax is 2.25% lower (see Figure 2) as minerals scarcity is not correctly taken into account when planning climate policy. In our baseline simulation, mineral scarcity constrains the economy substantially, welfare is much lower than expected by the regulator: the unexpected cost of mineral scarcity is equivalent ot a permanent fall of the electricity consumption path by 23.7%. Overall, public intervention implies a lower carbon tax and a higher FIP; it lasts longer and is less effective than it could (without myopia).

4 Recycling minerals

If CRM play an important role for the energy transition, recycling them represents a potentially interesting opportunity. Denote by $\alpha(t) \in [0, 1]$ the rate of recycling of the depreciated green capital $\delta Y(t)$. Non recycled minerals accumulate in a unrecoverable stock, a stock that is assumed not to induce any social damage. Recycling is costly. The associated cost function is $R(\alpha(t), \delta Y(t))$, assumed increasing and convex in the first argument, and increasing in the scale of the treated secondary stock $\delta Y(t)$. We further assume away any scale (dis)economies in recycling

$$R(\alpha, \delta Y) \equiv r(\alpha(t))\delta Y(t) \text{ with } r(\alpha) \geq 0,\ r'(\alpha) \geq 0 \text{ and } r''(\alpha) \geq 0 \tag{76}$$

Moreover, since we want to focus on plausible cases where recycling is limited, we also assume that the cost of perfect recycling (i.e. $\alpha(t) = 1$) is larger than the cost of using the backstop technology instead of recycled minerals to invest in green capital:

$$r'(1) > \nu/\delta \tag{77}$$

When using specific functional forms, in the sub-case of logarithmic utility and quadratic costs, we specify the quadratic function:

$$r(\alpha) \equiv r_1\alpha + \frac{r_2}{2}\alpha^2 \tag{78}$$

and thus assume: $r_1 + r_2 > \nu/\delta$.

In this section we first study the optimization problem in a centralized economy. We then consider potential market failure in the recycling activity, and study targeted public intervention the decentralized economy.

4.1 Optimal energy transition with minerals recycling

Planner's problem:

$$\max \int_0^\infty e^{-\rho t}[u(e(t)) - C(I(t)) - \nu b(t) - r(\alpha(t))\delta Y(t)]dt \tag{79}$$

$$\begin{gathered}
e(t) = x(t) + \phi Y(t) \\
I(t) = m(t) + b(t) + \alpha(t)\delta Y(t) \\
\dot{X}(t) = \varepsilon x(t) \\
\dot{Y}(t) = I(t) - \delta Y(t) \\
\dot{M}(t) = -m(t) \\
X(t) \leq \overline{X}, \quad x(t) \geq 0, \quad M(t) \geq 0, \quad m(t) \geq 0, \quad b(t) \geq 0 \\
0 \leq \alpha(t) \leq 1 \\
X(0) = X_0 \geq 0, \quad Y(0) = Y_0 \geq 0 \text{ and } M(0) = M_0 \geq 0 \text{ given}
\end{gathered}$$

The current value Hamiltonian associated to the social planner's program (79) is:

$$\mathcal{H}(.) = u(x + \phi Y) - C(m + b + \alpha\delta Y) - \nu b - r(\alpha)\delta Y - \lambda\varepsilon x + \mu(m + b - (1-\alpha)\delta Y) + \zeta(-m)$$

where λ is the shadow price of carbon, that is the carbon value, μ is the shadow price of green capital and ζ is the shadow price of the stock of minerals.

Introducing Lagrange multipliers for the different inequality constraints allows us to write the Lagrange function as:

$$\mathcal{L}(.) = \mathcal{H}(.) + \omega_x x + \omega_m m + \omega_b b + \omega_\alpha^0 \alpha + \omega_\alpha^1(1-\alpha) + \omega_X(\bar{X} - X) + \omega_M M$$

First order optimality conditions are:

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \iff u'(e) - \lambda\varepsilon + \omega_x = 0 \iff u'(e) = \varepsilon\lambda - \omega_x \tag{80}$$

$$\frac{\partial \mathcal{L}}{\partial m} = 0 \iff -C'(I) + \mu - \zeta + \omega_m = 0 \iff C'(I) = \mu - \zeta + \omega_m \tag{81}$$

$$\frac{\partial \mathcal{L}}{\partial b} = 0 \iff -C'(I) - \nu + \mu + \omega_b = 0 \iff C'(I) + \nu = \mu + \omega_b \tag{82}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = 0 \iff -C'(I)\delta Y - r'(\alpha)\delta Y + \mu\delta Y + \omega_\alpha^0 - \omega_\alpha^1 = 0 \iff C'(I) + r'(\alpha) = \mu + \frac{\omega_\alpha^0 - \omega_\alpha^1}{\delta Y} \tag{83}$$

$$-\frac{\partial \mathcal{L}}{\partial X} = \omega_X = -\dot{\lambda} + \rho\lambda \iff \dot{\lambda} = \rho\lambda - \omega_X \tag{84}$$

$$-\frac{\partial \mathcal{L}}{\partial Y} = -\phi u'(e) + C'(I)\alpha\delta + r(\alpha)\delta + (1-\alpha)\delta\mu = \dot{\mu} - \rho\mu \iff \dot{\mu} = (\rho + (1-\alpha)\delta)\mu - \phi u'(e) + \delta(C'(I)\alpha + r(\alpha)) \tag{85}$$

$$-\frac{\partial \mathcal{L}}{\partial M} = -\omega_M = \dot{\zeta} - \rho\zeta \iff \dot{\zeta} = \rho\zeta - \omega_M \tag{86}$$

First phase: $x > 0, m > 0, b = 0, \alpha = 0 \Rightarrow \omega_x = \omega_m = \omega_\alpha^1 = 0, \omega_b > 0, \omega_\alpha^0 > 0$. Before ceiling and before mineral scarcity is binding: $\omega_X = \omega_M = 0$.

Eq. (80) yields;

$$u'(x + \phi Y) = \varepsilon\lambda \tag{87}$$

The marginal utility of electricity consumption is equal in this phase to the carbon value, meaning that fossil is used.

Eq. (81) yields;

$$C'(m) = \mu - \zeta \iff C'(m) + \zeta = \mu \tag{88}$$

The total marginal cost of using minerals to invest in green capital (marginal investment cost plus scarcity rent, remember that there is no extraction cost) is equal to the value of green capital. Relevant iff $\mu > \zeta$, i.e. the value of green capital is higher than the value of the mineral stock under the ground.

Moreover, according to (82), $C'(m) + \nu > \mu$. Then we necessarily have in this phase $\zeta < \nu$: the scarcity rent of the mineral stock is smaller than the marginal cost of the backstop, which explains why the backstop is not used.

And according to (83), $C'(m) + r'(0) > \mu$. Then we necessarily have in this phase $\zeta < r'(0)$, which explains why there is no recycling.

Eqs (84) and (86): Hotelling on the two scarcity rents (fossil and minerals): $\lambda(t) = \lambda(0)e^{\rho t}$ and $\zeta(t) = \zeta(0)e^{\rho t}$.

Evolution of the shadow price and the stock of green capital:

$$\begin{cases} \dot{\mu} = (\rho + \delta)\mu - \phi\varepsilon\lambda \\ \dot{Y} = C'^{-1}(\mu - \zeta) - \delta Y \end{cases} \tag{89}$$

We study here the most relevant case where the ceiling is reached before the scarcity of the mineral input becomes binding. Then the first phase ends at date T_X, such that $x(t) = 0 \quad \forall t \geq T_X$, and $X(T_X) = \bar{X}$. T_X is defined by

$$\boxed{u'(\phi Y(T_X)) = \varepsilon\lambda(T_X)}$$

and we have

$$\varepsilon \int_0^{T_X} x(t)dt = \bar{X} \tag{90}$$

After T_X, clean phase with no fossil, investment in green capital using the mineral resource and not yet the backstop, supposed to be very expensive, and no recycling, since we'll also suppose that recycling is very expensive.

Second phase: $x = 0, m > 0, b = 0, \alpha = 0 \Rightarrow \omega_x > 0, \omega_m = \omega_\alpha^1 = 0, \omega_b > 0, \omega_\alpha^0 > 0$. Before mineral scarcity is binding: $\omega_M = 0$.

We still have (88), and therefore $\mu > \zeta$, as well as $C'(m) + \nu > \mu$ and $C'(m) + r'(0) > \mu$. Therefore $\zeta < \nu$ and $\zeta < r'(0)$.

Dynamic system:

$$\begin{cases} \dot{\mu} = (\rho + \delta)\mu - \phi u'(\phi Y) \\ \dot{Y} = C'^{-1}(\mu - \zeta) - \delta Y \end{cases} \tag{91}$$

Two possibilities at this stage: either $\nu > r'(0)$ or the opposite. In the first case, the second phase ends with the economy starting recycling, and not using the backstop yet. In the second case it starts using the backstop. We choose in the following to study the first case, which seems more relevant. We therefore make the assumption $\boxed{\nu > r'(0)}$. Then phase 2 stops at T_α such that

$$\boxed{\zeta(T_\alpha) = r'(0)}$$

Third phase: $x = 0, , b = 0, m > 0, \alpha > 0 \Rightarrow \omega_x > 0, \omega_b > 0, \omega_m = \omega_\alpha^0 = \omega_\alpha^1 = 0$. Before mineral scarcity is binding: $\omega_M = 0$.

Eqs (81) and (83) respectively read:

$$C'(I) = \mu - \zeta \qquad \text{with} \quad I = m + \alpha\delta Y \tag{92}$$

$$C'(I) + r'(\alpha) = \mu \tag{93}$$

and we still have $C'(I) + \nu > \mu$ since b is not used. Eqs (92) and (93) yield:

$$r'(\alpha) = \zeta \iff \alpha = r'^{-1}(\zeta)$$

Therefore in this phase the marginal cost of recycling is equal to the scarcity rent of minerals. It increases all along this phase, which means that α increases.

Dynamic system:

$$\begin{cases} \dot{\mu} = (\rho + \delta)\mu - \phi u'(\phi Y) - \delta\left(\zeta r'^{-1}(\zeta) - r(r'^{-1}(\zeta))\right) \\ \dot{Y} = C'^{-1}(\mu - \zeta) - \delta Y \end{cases} \tag{94}$$

This phase may end either because α reaches its maximum value (here equal to 1, but it would be better to make the assumption that there is a technological upper bound $\bar{\alpha} < 1$ to recycling possibilities), or because the scarcity rent of minerals reaches the marginal cost of the backstop. We study the second of these case, and for that make the assumption $\boxed{r'(1) > \nu}$. Then phase 3 ends at date T_M defined by

$$\boxed{\zeta(T_M) = r'(\alpha(T_M)) = \nu}$$

Then

$$C'(I(T_M)) + \nu = \mu(T_M)$$

with $I(T_M) = m(T_M^-) + \alpha(T_M)\delta Y(T_M) = b(T_M^+) + \alpha(T_M)\delta Y(T_M)$. and

$$\int_0^{T_M} m(t)dt = M_0 \tag{95}$$

Fourth phase: $x = 0, m = 0, b > 0 \Rightarrow \omega_x > 0, \omega_m > 0, \omega_b = 0$. Moreover, $\alpha = \alpha(T_M) = \alpha^*$.

Dynamic system:

$$\begin{cases} \dot{\mu} = (\rho + \delta)\mu - \phi u'(\phi Y) - \delta\left(\alpha^* r'(\alpha^*) - r(\alpha^*)\right) \\ \dot{Y} = C'^{-1}\left(\mu - \nu\right) - \delta Y \end{cases} \tag{96}$$

Saddle-path.

Steady state defined by:

$$\begin{cases} (\rho + \delta)\mu^* = \phi u'(\phi Y^*) + \delta\left(\alpha^* r'(\alpha^*) - r(\alpha^*)\right) \\ C'^{-1}\left(\mu^* - \nu\right) = \delta Y^* \end{cases} \tag{97}$$

4.1.1 Sub-case with quadratic costs and logarithmic utility

Consider

$$u(e(t)) = \gamma \ln e(t)$$

$$C(I) = c_1 I + \frac{c_2}{2} I^2 \quad \Rightarrow \quad C'(I) = c_1 + c_2 I, \quad C'(0) = c_1$$

$$r(\alpha) = r_1\alpha + \frac{r_2}{2}\alpha^2 \quad \Rightarrow \quad r'(\alpha) = r_1 + r_2\alpha, \quad r'(0) = r_1, \quad r'(1) = r_1 + r_2$$

Then we have

$$\alpha^* = \frac{\nu - r_1}{r_2} \tag{98}$$

$$\forall t \in (0, T_X) \quad \begin{cases} \dot{\mu} = (\rho + \delta)\mu - \phi\varepsilon\lambda \\ \dot{Y} = \frac{1}{c_2}(\mu - \zeta - c_1) - \delta Y \end{cases} \tag{99}$$

$$\forall t \in [T_X, T_\alpha) \quad \begin{cases} \dot{\mu} = (\rho + \delta)\mu - \frac{\gamma}{Y} \\ \dot{Y} = \frac{1}{c_2}(\mu - \zeta - c_1) - \delta Y \end{cases} \tag{100}$$

$$\forall t \in [T_\alpha, T_M) \quad \begin{cases} \dot{\mu} = (\rho + \delta)\mu - \frac{\gamma}{Y} - \delta\frac{(\zeta - r_1)^2}{2r_2} \\ \dot{Y} = \frac{1}{c_2}(\mu - \zeta - c_1) - \delta Y \end{cases} \tag{101}$$

$$\forall t \geq T_M \quad \begin{cases} \dot{\mu} = (\rho + \delta)\mu - \frac{\gamma}{Y} - \delta\frac{(\nu - r_1)^2}{2r_2} \\ \dot{Y} = \frac{1}{c_2}(\mu - \nu - c_1) - \delta Y \end{cases} \tag{102}$$

$$\mu^* = \frac{1}{2}\left(\nu + c_1 + \frac{\delta r_2(\alpha^*)^2}{2(\rho+\delta)}\right)\left[1 + \left(1 + 4\delta\frac{\gamma c_2 - (\nu+c_1)(\alpha^*)^2 r_2/2}{(\rho+\delta)\left(\nu + c_1 + \frac{\delta r_2(\alpha^*)^2}{2(\rho+\delta)}\right)^2}\right)^{1/2}\right] \tag{103}$$

$$Y^* = \frac{\mu^* - (\nu + c_1)}{\delta c_2} \tag{104}$$

$$b^* = (1-\alpha^*)\delta Y^* \tag{105}$$

The restrictions are

$$\lambda(0) < \frac{\gamma}{\varepsilon\phi Y_0} \quad \Leftrightarrow \quad x(0) > 0, \quad T_X > 0 \tag{106}$$

$$\zeta(0) < \mu(0) - c_1 \quad \Leftrightarrow \quad m(0) > 0, \quad T_M > 0 \tag{107}$$

$$\zeta(0) < r_1 \quad \Leftrightarrow \quad \alpha(0) = 0 \tag{108}$$

$$\nu < r_1 + r_2 \quad \Leftrightarrow \quad \alpha^* < 1 \tag{109}$$

$$c_1 < \nu \quad \Leftrightarrow \quad T_m > 0 \tag{110}$$

$$r_1 < \nu \quad \Leftrightarrow \quad T_\alpha < T_m \tag{111}$$

Table 3 reports the parameters' values used to conduct some comparative dynamics exercises. Optimal dates for the succession of phases in case of a more stringent climate policy, a bad surprise on CRM stock or a higher recycling cost are presented in table 4.

ϕ	ρ	δ	ε	γ	ν	c_1	c_2	α^*	r_1	r_2	Y_0	M_0	$\overline{X}$
0.9	0.03	0.018	1	1.8	5	0.15	20	0.2	4	5	1	6	37.4

Table 3: Parameters for the baseline simulation.

A more stringent climate policy fastens the whole process as fossil fuels are abandoned sooner, recycling starts earlier and there is a quicker exhaustion of CRM. A smaller initial stock of CRM delays the fossil phase-out but speeds up recycling and CRM exhaustion. Finally, a more expensive recycling process starts later and ends at a lower steady-state rate but leads to a quicker CRM exhaustion.

	T_X	T_α	T_M	α^*
Reference	16.0	22.5	30.0	20%
More stringent climate policy ($0.8\bar{X}$)	14.2	22.0	29.4	20%
Bad surprise on CRM ($0.9M_0$)	16.2	19.2	26.6	20%
Higher recycling cost ($r_1 = 4.5$)	16.0	26.3	29.8	10%

Table 4: Comparative dynamics.

4.2 Alternative successions for the phases

4.2.1 Low recycling costs: $T_\alpha < T_X < T_M$

First phase: $x > 0, m > 0, b = 0, \alpha = 0 \Rightarrow \omega_x = \omega_m = \omega_\alpha^1 = 0, \omega_b > 0, \omega_\alpha^0 > 0$. Before ceiling and before mineral scarcity is binding: $\omega_X = \omega_M = 0$.

Relevant iff $\mu > \zeta$, i.e. the value of green capital is higher than the value of the mineral stock under the ground. We necessarily have in this phase $\zeta < \nu$ (the scarcity rent of the mineral stock is smaller than the marginal cost of the backstop, which explains why the backstop is not used) and $\zeta < r'(0)$ (which explains why there is no recycling). Evolution of the shadow price and the stock of green capital:

$$\left\{ \begin{array}{l} \dot{\mu} = (\rho + \delta)\mu - \phi\varepsilon\lambda \\ \dot{Y} = C'^{-1}(\mu - \zeta) - \delta Y \end{array} \right. \tag{112}$$

We study here the case where recycling starts before the ceiling is reached.

In addition we make the assumption $\boxed{\nu > r'(0)}$ such that recycling starts before the backstop.

Then phase 1 stops at T_α such that

$$\boxed{\zeta(T_\alpha) = r'(0)}$$

Second phase: $x > 0, m > 0, b = 0, \alpha > 0 \Rightarrow \omega_x = \omega_m = 0 = \omega_\alpha^1 = \omega_\alpha^0 = 0, \omega_b > 0$. Before ceiling and before mineral scarcity is binding: $\omega_M = 0$.

We still have $\mu > \zeta$, as well as $\zeta < \nu$. In addition, $r'(\alpha) = \zeta \iff \alpha = r'^{-1}(\zeta)$.

Dynamic system:

$$\left\{ \begin{array}{l} \dot{\mu} = (\rho + \delta)\mu - \phi\epsilon\lambda_0 e^{\rho t} - \delta(\zeta r'^{-1}(\zeta) - r(r'^{-1}(\zeta))) \\ \dot{Y} = C'^{-1}(\mu - \zeta) - \delta Y \end{array} \right. \tag{113}$$

Phase 2 stops at T_X such that $x(t) = 0 \quad \forall t \geq T_X$, and $X(T_X) = \bar{X}$. T_X is defined by

$$\boxed{u'(\phi Y(T_X)) = \varepsilon\lambda(T_X)}$$

and we have

$$\varepsilon \int_0^{T_X} x(t)dt = \bar{X} \tag{114}$$

that provides λ_0 After T_X, clean phase with no fossil, investment in green capital using the mineral resource and not yet the backstop, supposed to be very expensive, but goes on with recycling. Phases 3 and 4 are similar as in the case where recyling starts later.

Quadratic costs and logarithmic utility

Then we have

$$\alpha^* = \frac{\nu - r_1}{r_2} \tag{115}$$

$$\forall t \in (0, T_\alpha) \quad \left\{ \begin{array}{l} \dot{\mu} = (\rho + \delta)\mu - \phi\varepsilon\lambda \\ \dot{Y} = \frac{1}{c_2}(\mu - \zeta - c_1) - \delta Y \end{array} \right. \tag{116}$$

$$\forall t \in [T_\alpha, T_X) \quad \left\{ \begin{array}{l} \dot{\mu} = (\rho + \delta)\mu - \phi\varepsilon\lambda - \delta\frac{(\zeta - r_1)^2}{2r_2} \\ \dot{Y} = \frac{1}{c_2}(\mu - \zeta - c_1) - \delta Y \end{array} \right. \tag{117}$$

$$\forall t \in [T_X, T_M) \quad \left\{ \begin{array}{l} \dot{\mu} = (\rho + \delta)\mu - \frac{\gamma}{Y} - \delta\frac{(\zeta - r_1)^2}{2r_2} \\ \dot{Y} = \frac{1}{c_2}(\mu - \zeta - c_1) - \delta Y \end{array} \right. \tag{118}$$

$$\forall t \geq T_M \quad \begin{cases} \dot{\mu} = (\rho+\delta)\mu - \frac{\gamma}{Y} - \delta\frac{(\nu-r_1)^2}{2r_2} \\ \dot{Y} = \frac{1}{c_2}(\mu-\nu-c_1) - \delta Y \end{cases} \tag{119}$$

$$\mu^* = \frac{1}{2}\left(\nu + c_1 + \frac{\delta r_2(\alpha^*)^2}{2(\rho+\delta)}\right)\left[1 + \left(1 + 4\delta\frac{\gamma c_2 - (\nu+c_1)(\alpha^*)^2 r_2/2}{(\rho+\delta)\left(\nu+c_1+\frac{\delta r_2(\alpha^*)^2}{2(\rho+\delta)}\right)^2}\right)^{1/2}\right] \tag{120}$$

$$Y^* = \frac{\mu^* - (\nu+c_1)}{\delta c_2} \tag{121}$$

The restrictions are

$$\lambda(0) < \frac{\gamma}{\varepsilon\phi Y_0} \quad \Leftrightarrow \quad x(0) > 0 \quad T_X > 0 \tag{122}$$

$$\zeta(0) < \mu(0) - c_1 \quad \Leftrightarrow \quad m(0) > 0, \quad T_M > 0 \tag{123}$$

$$\zeta(0) < r_1 \quad \Leftrightarrow \quad \alpha(0) = 0 \tag{124}$$

$$\nu < r_1 + r_2 \quad \Leftrightarrow \quad \alpha < 1 \tag{125}$$

$$r_1 < \nu \quad \Leftrightarrow \quad T_\alpha < T_M \tag{126}$$

4.2.2 Large carbon budget : $T_\alpha < T_M < T_X$

The first phase is unchanged. It stops at at T_α as in section 4.2.1

Second phase: $x > 0, m > 0, b = 0, \alpha > 0 \Rightarrow \omega_x = \omega_m = 0 = \omega_\alpha^1 = \omega_\alpha^0 = 0, \omega_b > 0$. Before ceiling and before mineral scarcity is binding: $\omega_M = 0$.

We still have $\mu > \zeta$, as well as $\zeta < \nu$. In addition, $r'(\alpha) = \zeta \iff \alpha = r'^{-1}(\zeta)$.

Dynamic system:

$$\begin{cases} \dot{\mu} = (\rho+\delta)\mu - \phi\epsilon\lambda_0 e^{\rho t} - \delta(\zeta r'^{-1}(\zeta) - r(r'^{-1}(\zeta))) \\ \dot{Y} = C'^{-1}(\mu - \zeta) - \delta Y \end{cases} \tag{127}$$

Phase 2 stops at T_M such that the scarcity rent of minerals reaches the marginal cost of the backstop (we assume again that $r'(1) > \nu$):

$$\zeta(T_M) = \nu = \mu - C'(I(T_M)) = r'(\alpha(T_M))$$

with $I(T_M) = m(T_M^-) + \alpha(T_M)\delta Y(T_M) = b(T_M^+) + \alpha(T_M)\delta Y(T_M)$. and

$$\int_0^{T_M} m(t)dt = M_0 \tag{128}$$

that provides ζ_0.

After T_M, phase with fossil, investment in green capital using the backstop and goes on with recycling.

Third phase: $x > 0, m = 0, b > 0, \alpha > 0 \Rightarrow \omega_x = \omega_\alpha^1 = \omega_\alpha^0 = \omega_b = 0, \omega_m > 0$. Before ceiling: $\omega_X = 0$.

$r'(\alpha) = \nu \iff \alpha = r'^{-1}(\nu)$.

Dynamic system:

$$\left\{ \begin{array}{l} \dot{\mu} = (\rho+\delta)\mu - \phi\epsilon\lambda_0 e^{\rho t} - \delta[\nu r'^{-1}(\nu) - r(r'^{-1}(\nu))] \\ \dot{Y} = C'^{-1}(\mu - \nu) - \delta Y \end{array} \right. \tag{129}$$

Note that this sytem can be solved analytically.
Phase 3 stops at T_X such that $x(t) = 0 \quad \forall t \geq T_X$, and $X(T_X) = \bar{X}$. T_X is defined by

$$\boxed{u'(\phi Y(T_X)) = \varepsilon\lambda(T_X)}$$

and we have

$$\varepsilon \int_0^{T_X} x(t)dt = \bar{X} \tag{130}$$

that provides λ_0 After T_X, clean phase with no fossil, investment in green capital using the backstop and goes on with recycling.
Phase 4 is similar as in the other cases.

Quadratic costs and logarithmic utility

Then we have

$$\alpha^* = \frac{\nu - r_1}{r_2} \tag{131}$$

$$\forall t \in (0, T_\alpha) \quad \left\{ \begin{array}{l} \dot{\mu} = (\rho+\delta)\mu - \phi\varepsilon\lambda \\ \dot{Y} = \frac{1}{c_2}(\mu - \zeta - c_1) - \delta Y \end{array} \right. \tag{132}$$

$$\forall t \in [T_\alpha, T_M) \quad \left\{ \begin{array}{l} \dot{\mu} = (\rho+\delta)\mu - \phi\varepsilon\lambda - \delta\frac{(\zeta - r_1)^2}{2r_2} \\ \dot{Y} = \frac{1}{c_2}(\mu - \zeta - c_1) - \delta Y \end{array} \right. \tag{133}$$

$$\forall t \in [T_M, T_X) \quad \left\{ \begin{array}{l} \dot{\mu} = (\rho+\delta)\mu - \phi\varepsilon\lambda - \delta\frac{(\nu - r_1)^2}{2r_2} \\ \dot{Y} = \frac{1}{c_2}(\mu - \nu - c_1) - \delta Y \end{array} \right. \tag{134}$$

$$\forall t \geq T_X \quad \left\{ \begin{array}{l} \dot{\mu} = (\rho+\delta)\mu - \frac{\gamma}{Y} - \delta\frac{(\nu - r_1)^2}{2r_2} \\ \dot{Y} = \frac{1}{c_2}(\mu - \nu - c_1) - \delta Y \end{array} \right. \tag{135}$$

$$\mu^* = \frac{1}{2}\left(\nu + c_1 + \frac{\delta r_2(\alpha^*)^2}{2(\rho+\delta)}\right)\left[1 + \left(1 + 4\delta\frac{\gamma c_2 - (\nu + c_1)(\alpha^*)^2 r_2/2}{(\rho+\delta)\left(\nu + c_1 + \frac{\delta r_2(\alpha^*)^2}{2(\rho+\delta)}\right)^2}\right)^{1/2}\right] \tag{136}$$

$$Y^* = \frac{\mu^* - (\nu + c_1)}{\delta c_2} \tag{137}$$

The restrictions are

$$\lambda(0) < \frac{\gamma}{\varepsilon\phi Y_0} \quad \Leftrightarrow \quad x(0) > 0 \tag{138}$$

$$\zeta(0) < \mu(0) - c_1 \quad \Leftrightarrow \quad m(0) > 0, T_M > 0 \tag{139}$$

$$\zeta(0) < r_1 \quad \Leftrightarrow \quad \alpha(0) = 0 \tag{140}$$

$$\nu < r_1 + r_2 \quad \Leftrightarrow \quad \alpha < 1 \tag{141}$$

$$r_1 < \nu \quad \Leftrightarrow \quad T_\alpha < T_M \tag{142}$$

4.2.3 Expensive recycling: $T_X < T_M < T_\alpha$

The first phase is the same as in the main model. It ends at T_X when the carbon budget is exhausted.

Second phase: $x = 0, m > 0, b = 0, \alpha = 0 \Rightarrow \omega_x > 0, \omega_m = \omega_\alpha^1 = 0, \omega_b > 0, \omega_\alpha^0 > 0$. Before mineral scarcity is binding: $\omega_M = 0$. We still have $\mu > \zeta$, as well as $\zeta < \nu$ and $\zeta < r'(0)$.

Dynamic system:

$$\begin{cases} \dot{\mu} = (\rho + \delta)\mu - \phi u'(\phi Y) \\ \dot{Y} = C'^{-1}(\mu - \zeta) - \delta Y \end{cases} \tag{143}$$

We assume that the second phase ends with the economy starting using the backstop and not recycling yet. We therefore make the assumption $\boxed{\nu < r'(0)}$. In fact, in such a case, it is never optimal to start recycling as it will always be more expensive than the backstop. When both are used, equations (83) and (82) imply that $r'(\alpha) = \nu$, that is not possible as $\nu < r'(0)$ and $r''(.) > 0$. Then phase 2 stops at T_M such that

$$\zeta(T_M) = \nu = \mu - C'(I(T_M))$$

with $I(T_M) = m(T_M^-) = b(T_M^+)$. and

$$\int_0^{T_M} m(t)dt = M_0 \tag{144}$$

that provides ζ_0.

After T_M, the third phase starts with neither fossil nor minerals, investment in green capital using the backstop : it is similar to the third phase in the absence absence of recycling possibility, see section 2.

5 Concluding remarks

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A Appendix

A.1 Constant carbon tax and FIP with strict balanced budget, isoelastic utility

Consider now the case where the regulator credibly commits to charging a constant unit tax on carbon emissions, denoted τ, and to use the tax revenue to subsidize the purchase of electricity produced from green capital, as with feed-in-premiums (FIP), at rate $\sigma(t)$. Moreover, we assume strict budget balance, i.e. no unit tax on consumption of electricity, $\theta(t) = 0$, and thus at each date

$$\tau\varepsilon x(t) = \sigma(t)\phi Y(t) \tag{145}$$

Substituting for $x(t)$ from the definition of electricity consumption $e(t) = x(t) + \phi Y(t)$, and rearranging,:

$$\sigma(t) = \left(\frac{e(t)}{\phi Y(t)} - 1\right)\tau\varepsilon \in \left[0, \tau\frac{\varepsilon x(0)}{\phi Y_0}\right] \tag{146}$$

The conditions on the mineral market are unchanged.

We consider now an isoelastic utility function:

$$u(e_t) \equiv \gamma \frac{e(t)^{1-\omega}}{1-\omega} \quad \omega \geq 0 \tag{147}$$

The FOC of the consumer problem hands the electricity demand function

$$e(t) = \left(\frac{\gamma}{P_e(t)}\right)^{\frac{1}{\omega}} \quad \text{or} \quad P_e(t) = \gamma(e(t))^{-\omega} \tag{148}$$

The FOC for fossil resource use by electricity producer is

$$P_e(t) = \tau\varepsilon \tag{149}$$

For the electricity producer the value of green capital evolves according to

$$\dot{\mu}_d(t) = (r+\delta)\,\mu(t)_d - \left(\gamma(e(t))^{-\omega} + \sigma(t)\right)\phi \tag{150}$$

Combining the first two of the three equations above, we note that electricity consumption is constant as long a fossil resources are used for electricity production:

$$e(t) = \left(\frac{\gamma}{\tau\varepsilon}\right)^{\frac{1}{\omega}} \tag{151}$$

and using the definition of $e(t)$

$$x(t) = \left(\frac{\gamma}{\tau\varepsilon}\right)^{\frac{1}{\omega}} - \phi Y(t) \tag{152}$$

Hence, as the stock of green capital increases, fossil resource use decreases from $x(0) = (\gamma/(\tau\varepsilon))^{\frac{1}{\omega}} - \phi Y_0$ (notice that $\tau < \gamma/\left(\varepsilon(\phi Y_0)^{\omega}\right) \Leftrightarrow x(0) > 0$), down to $x(T_X) = 0$ when $Y(T_X) = (1/\phi)(\gamma/(\tau\varepsilon))^{\frac{1}{\omega}}$ (notice that $\tau > \gamma/\left(\varepsilon(\phi Y^*)^{\omega}\right) \Leftrightarrow Y(T_X) < Y^*$).

Substituting for $e(t)$, then for $x(t)$, into the FIP $\sigma(t)$ from the balanced budget

$$\begin{aligned}
\sigma(t) &= \left(\frac{e(t)}{\phi Y(t)} - 1\right)\tau\varepsilon \\
&= \left(\frac{\left(\frac{\gamma}{\tau\varepsilon}\right)^{\frac{1}{\omega}}}{\phi Y(t)} - 1\right)\tau\varepsilon \\
&= \frac{\gamma^{\frac{1}{\omega}}}{\phi Y(t)}\,(\tau\varepsilon)^{\frac{\omega-1}{\omega}} - \tau\varepsilon
\end{aligned}$$

Using this expression to substitute for $\sigma(t)$ into the Euler equation for the value of green capital, as well as for the electricity price from the FOCs of the consumer and electricity producer problems, we have that $\forall t \in [0, T_X)$

$$\begin{aligned}\dot{\mu}_d(t) &= (r+\delta)_d\,\mu(t) - \left(\tau\varepsilon + \frac{\gamma^{\frac{1}{\omega}}}{\phi Y(t)}\,(\tau\varepsilon)^{\frac{\omega-1}{\omega}} - \tau\varepsilon\right)\phi \\ &= (r+\delta)\,\mu_d(t) - \left(\frac{\tau\varepsilon}{\gamma}\right)^{\frac{\omega-1}{\omega}}\frac{\gamma}{Y(t)} \end{aligned} \tag{153}$$

In this case, the policy affects the trajectory. The system of ordinary differential equations is specific to each of the three phases :

$$\forall t \in [0, T_X) \quad \begin{cases} \dot{\mu}_d(t) = (r+\delta)\mu_d(t) - \left(\frac{\tau\varepsilon}{\gamma}\right)^{\frac{\omega-1}{\omega}}\frac{\gamma}{Y(t)} \\ \dot{Y}(t) = \frac{1}{c_2}\left(\mu_d(t) - P_m(0)e^{rt} - c_1\right) - \delta Y(t) \end{cases} \tag{154}$$

$$\text{if } T_X < T_M, \quad \forall t \in [T_X, T_M) \quad \begin{cases} \dot{\mu}_d(t) = (r+\delta)\mu_d(t) - \frac{\gamma(\phi^{1-\omega})}{Y(t)^\omega} \\ \dot{Y}(t) = \frac{1}{c_2}\left(\mu_d(t) - P_m(0)e^{rt} - c_1\right) - \delta Y(t) \end{cases} \tag{155}$$

$$\forall t \geq \max\{T_M, T_X\} \quad \begin{cases} \dot{\mu}_d(t) = (r+\delta)\mu_d(t) - \frac{\gamma(\phi^{1-\omega})}{Y(t)^\omega} \\ \dot{Y}(t) = \frac{1}{c_2}\left(\mu_d(t) - \nu - c_1\right) - \delta Y(t) \end{cases} \tag{156}$$

The system if specified for the time sequence $T_x = T_m = 0 < T_X < T_M$, while if $T_x = T_m = 0 < T_X > T_M$ the first system applies for $t < T_X$ and the last one for $t \geq T_X$.

Initial conditions $Y(0) = Y_0$, $X(0) = X_0$, $P_m(0)$ pinned down by the resulting path of $m(t)$ and the initial condition M_0.
Exogenous policy variable: τ.
Endogenous variables: T_X, T_M, $\mu_d(0)$, $X(T_X)$.

The steady state is independent of the policy:

$$\begin{cases} \mu_d^* : \quad \frac{\mu_d^*}{\delta c_2} - \left(\frac{\gamma}{\rho+\delta}\right)^{\frac{1}{\omega}}\phi^{\frac{1-\omega}{\omega}}(\mu_d^*)^{-\frac{1}{\omega}} - \frac{\nu+c_1}{\delta c_2} = 0 \\ Y^* = \frac{\mu_d^* - \nu - c_1}{c_2\delta} \end{cases} \tag{157}$$

There exists a unique μ_d^*, since the function defining it increases monotonically from $-\infty$ for $\mu_d^* = 0$, up to $+\infty$ for $\mu_d^* = \infty$.

We find that the policy affects the use of fossil resources by reducing it linearly through a direct effect, as in the case of logarithmic utility. However, the carbon tax and FIP now also influence the evolution of the value of green capital, and in opposite directions according to whether the elasticity of intertemporal substitution of electricity consumption is larger or smaller than unity.